# The Generalized Second Law of Thermodynamics of the Universe Bounded by the Event Horizon and Modified Gravity Theories

**Nairwita Mazumder · Subenoy Chakraborty**

**Abstract** In this paper, we investigate the validity of the generalized second law of thermodynamics of the universe bounded by the event horizon. Here we consider homogeneous and isotropic model of the universe filled with perfect fluid in one case and in another case holographic model of the universe has been considered. In the third case the matter in the universe is taken in the form of non-interacting two fluid system as holographic dark energy and dust. Here we study the above cases in the Modified gravity, $f(R)$ gravity.

**Keywords** Modified gravity theories · Thermodynamics

## 1 Introduction

After the discovery of black hole thermodynamics physicists start speculating about the inherent link between the black hole thermodynamics and Einstein's field equations. Jacobson first showed the link between general relativity and thermodynamics [1]. Einstein's equation were shown to be the consequences of the proportionality of entropy and horizon area which was first to outline the laws of thermodynamics in the field of a black hole by Bekenstein [2]. Later Padmanavan [3] derived the first law of thermodynamics on the apparent horizon starting from Einstein's equations on it. If we assume the universe as a thermodynamical system and consider that at the apparent horizon $R = R_A$, the Hawking temperature $T_A = \frac{1}{2\pi R_A}$ and the entropy $S_A = \frac{\pi R_A^2}{G}$, then it can be shown that the first law of thermodynamics and the Friedmann equations are equivalent [4].

At present there are various observational data which strongly suggest that the current expansion of the universe is accelerating [5–12]. Now the reason for this accelerating expansion can be described in two ways [13–20]. One is to introducing the dark energy having

N. Mazumder · S. Chakraborty (✉)
Department of Mathematics, Jadavpur University, Kolkata 32, India
e-mail: schakraborty@math.jdvu.ac.in

N. Mazumder
e-mail: nairwita15@gmail.com

negative pressure in the frame work of general relativity and other is to study a modified gravitational theory, such as $f(R)$ gravity [18–23] where the action term is described by an arbitrary function $f(R)$ of the scalar curvature $R$. In this modified theory of gravity, instead of Friedmann equations we have the modified Friedmann equations [24, 25] which includes the powers of Ricci Scalar and its time derivatives. In $f(R)$ gravity the expression of the entropy is also different from that of Einstein gravity namely, $S = f'(R)\frac{A}{4G}$ [24].

Similar to this situation the entropy area relation may have a logarithmic correction term i.e. $S = \frac{A}{4G} + \alpha \ln \frac{A}{4G}$, where $\alpha$ is a dimension less constant. Such a correction term is expected to be a generic one in any theory of quantum gravity. Due to this quantum correction, the Friedmann equations also get modified and are familiar in the form to loop quantum gravity [26–34]. In this modified gravity theory the big bang singularity can be described by a quantum bounce [35–37].

In the usual standard big bang model a cosmological event horizon does not exist. But for the accelerating universe dominated by dark energy, with equation of state $\omega_D \neq -1$, the cosmological event horizon separates from that of the apparent horizon. Wang et al. [38] have shown that by applying the usual definition of temperature and entropy as in apparent horizon to the cosmological event horizon, both first and second law of thermodynamics breaks down. They have argued that the first law is applicable to nearby states of local thermodynamic equilibrium but event horizon reflects the global features of space-time. The definition of thermodynamical quantities on the cosmological event horizon in the non-static universe may not be as simple as in the static space-time. Further, it is speculated that the region bounded by the apparent horizon may be considered as the Bekenstein system i.e. Bekenstein's entropy-mass bound ($S \leq 2\pi R_E$) and entropy-area bound ($S \leq \frac{A}{4}$) are obeyed in this region. Moreover, Bekenstein bounds are universal and all gravitationally stable special regions with weak self gravity should satisfy Bekenstein bounds and the corresponding thermodynamical system is termed as Bekenstein system. But in the case when we are dealing with the universe is bounded by the event horizon we cannot exactly say whether it can be considered as a Bekenstein system or not. Generally the universe bounded by the event horizon is not considered as a Bekenstein system [38].

In this paper we try to find the necessary conditions for the validity of the generalized second law of thermodynamics on the event horizon both modified gravity with logarithmic correction in entropy-area relation (MGLC) and in $f(R)$ gravity. Here we have not assumed any explicit expression for entropy and temperature in the event horizon but have only assumed the validity of the first law of thermodynamics (which can be viewed as an energy conservation relation) on the event horizon.

The paper is organized as follows: in Sect. 2 we have taken the homogeneous and isotropic model of the universe filled with perfect fluid both in MGLC as well as in $f(R)$ gravity. In Sect. 3 we have considered the universe filled only with holographic dark energy. The holographic dark energy model [38–50] has been constructed in the light of holographic principle of quantum gravity theory. Then in Sect. 4, matter in the non-interacting combination of holographic dark energy and dust has been chosen for the universe. The conditions for the validity of the generalized second law of thermodynamics on the event horizon are determined both in MGLC and $f(R)$ gravity. Finally, the paper ends with concluding remarks in Sect. 5.

## 2 Generalized Second Law of Thermodynamics of the Universe Filled with Perfect Fluid

### 2.1 Modified Gravity with Logarithmic Correction in Entropy-Area Relation (MGLC)

In FRW model of the universe filled with perfect fluid with non-zero spatial curvature, the modified Friedmann equations in MGLC are given by [51]

$$\left[1 + \frac{\alpha G}{\pi}\left(H^2 + \frac{k}{a^2}\right)\right]\left(\dot{H} - \frac{k}{a^2}\right) = -4\pi G(\rho + p), \tag{1}$$

$$\left(H^2 + \frac{k}{a^2}\right)\left[1 + \frac{\alpha G}{2\pi}\left(H^2 + \frac{k}{a^2}\right)\right] = \frac{8\pi G\rho}{3} \tag{2}$$

and the continuity equation is

$$\dot{\rho} + 3H(\rho + p) = 0. \tag{3}$$

Here $\alpha$ is a dimension less constant. The radius of the cosmological event horizon is given by

$$\tilde{r}_E = a \int_t^\infty \frac{dt}{a} = a \int_a^\infty \frac{da}{Ha^2}. \tag{4}$$

The amount of energy crossing the event horizon during time $dt$ is given by [4, 52]

$$-dE = 4\pi \tilde{r}_E^3 H(\rho + p)dt. \tag{5}$$

Now assuming the validity of the first law of thermodynamics at the event horizon (i.e. $-dE = T_E dS_E$) we have

$$dS_E = \frac{4\pi \tilde{r}_E^3 H(\rho + p)dt}{T_E} \tag{6}$$

where $S_E$ is the entropy of the event horizon and $T_E$ is the temperature on the event horizon.

To show the validity of the generalized second law of thermodynamics we start with Gibbs equation [53]

$$T_E dS_I = dE_I + pdV \tag{7}$$

where $S_I$ is the entropy of the matter bounded by the event horizon and $E_I$ is the energy of the matter distribution. Here for thermodynamical equilibrium, the temperature of the matter inside the event horizon is assumed to be same as on the event horizon i.e. $T_E$. Now, starting with

$$V = \frac{4\pi \tilde{r}_E^3}{3}, \qquad E_I = \frac{4\pi \tilde{r}_E^3 \rho}{3} \tag{8}$$

and the continuity equation (3), the Gibbs equation leads to

$$dS_I = -4\pi \frac{\tilde{r}_E^2}{T_E}(\rho + p)dt, \tag{9}$$

where we have used the variation of the radius of the event horizon [54] as

$$d\tilde{r}_E = (\tilde{r}_E H - 1)dt. \tag{10}$$

So combining (6) and (9) we get

$$\frac{d}{dt}(S_I + S_E) = 4\pi(\rho + p)\frac{\tilde{r}_E^2 H}{T_E}\left(\tilde{r}_E - \frac{1}{H}\right). \tag{11}$$

Thus the expression in (11) for the rate of change of total entropy show that validity of the second law of thermodynamics depends both on geometry as well as on the matter in the universe.

## 2.2 $f(R)$ Gravity

For the spatially flat FRW universe the modified Friedmann equations [25] in the $f(R)$ gravity are given as

$$8\pi\rho = \frac{f(R)}{2} - 3\left(\dot{H} + H^2 - H\frac{d}{dt}\right)f'(R), \tag{12}$$

$$8\pi(\rho + p) = -2\dot{H}f'(R) + H\frac{d}{dt}f'(R) - \frac{d^2}{dt^2}f'(R) \tag{13}$$

where the Ricci Scalar $R = 6\dot{H} + 12H^2$. The over dot indicates the derivative with respect to the co-moving time $t$. The continuity equation is same as (3).

Since the horizons are determined purely by the geometry, independent of the gravity theories so as before assuming the validity of the first law of thermodynamics the change in the horizon entropy is given by

$$dS_E = \frac{\tilde{r}_E^3 H}{2T_E}\left(-2\dot{H}f'(R) + H\frac{d}{dt}f'(R) - \frac{d^2}{dt^2}f'(R)\right)dt. \tag{14}$$

But this equation can be written also as (6). Now from the Gibbs equation the change in entropy of the matter inside the event horizon is given by

$$\begin{aligned}
dS_I &= \frac{1}{T_E}(dE_I + pdV) \\
&= \frac{1}{2T_E}\left[\left(-2\dot{H}f'(R) + H\frac{d}{dt}f'(R) - \frac{d^2}{dt^2}f'(R)\right)\tilde{r}_E^3\left(\tilde{r}_E - \frac{1}{H}\right)dt \right. \\
&\quad \left. + \frac{8}{3}\pi\tilde{r}_E^3 d\rho\right].
\end{aligned} \tag{15}$$

Using equation of continuity we get from the expression (13)

$$dS_I = -\frac{\tilde{r}_E^2}{2T_E}\left(-2\dot{H}f'(R) + H\frac{d}{dt}f'(R) - \frac{d^2}{dt^2}f'(R)\right)dt. \tag{16}$$

Thus combining (14), (15) and using modified Friedmann equations we get

$$\begin{aligned}
\frac{d}{dt}(S_I + S_E) &= \frac{\tilde{r}_E^2 H}{2T_E}\left(-2\dot{H}f'(R) + H\frac{d}{dt}f'(R) - \frac{d^2}{dt^2}f'(R)\right)\left(\tilde{r}_E - \frac{1}{H}\right) \\
&= 4\pi(\rho + p)\frac{\tilde{r}_E^2 H}{T_E}\left(\tilde{r}_E - \frac{1}{H}\right)
\end{aligned} \tag{17}$$

which is same as in the MGLC. So we can conclude that assuming the validity of the first law of thermodynamics, the criteria for holding the second law of thermodynamics on the event horizon is same for the above two modified gravity theories. Further, one may note that although for the calculations for $f(R)$ gravity in case B we have used the modified Friedmann equations but it is possible to do the calculations similar to case A without using modified Friedmann equations.

## 3 Generalized Second Law of Thermodynamics of the Universe Filled with Holographic Dark Energy

The holographic dark energy model is important for the present accelerating phase of the universe. Also in the present context, the choice of holographic dark energy is justified as an explicit expression for the radius of the event horizon can be obtained. Due to the complicated form of the field equations we consider the flat FRW model of the universe filled with holographic dark energy. Now choosing $8\pi G = 1$ we have the modified Friedmann equation in LQG as follows

$$\left(1 + \frac{\alpha}{8\pi^2}H^2\right)\dot{H} = -\frac{(\rho_D + p_D)}{2}, \tag{18}$$

$$H^2\left(1 + \frac{\alpha}{16\pi^2}H^2\right) = \frac{\rho_D}{3} \tag{19}$$

with same form of equation of continuity as (3).

The radius of the event horizon considering the holographic dark energy model [38] is

$$R_E = \frac{c}{H\sqrt{\Omega_D}}$$

where the density parameter has the expression $\Omega_D = \frac{\rho_D}{3H^2}$. The equation of state of the dark energy can be written as

$$\rho_D = \omega_D p_D$$

where $\omega_D$ is not necessarily a constant.

So from the holographic model a small variation of the radius of the horizon is given by [55]

$$dR_E = \frac{3}{2}R_E H(1 + \omega_D)dt. \tag{20}$$

Since the amount of energy crossing the horizon in time $dt$ does not depend on any particular gravity theory so from the first law of thermodynamics we get

$$\frac{dS_E}{dt} = 4\pi H\rho_D \frac{R_E^3}{T_E}(1 + \omega_D) \tag{21}$$

where $T_E$ is the temperature of the event horizon. Now to obtain the variation of the entropy of the fluid inside the event horizon we use as before the Gibbs equation (7) with

$$E_I = \frac{4}{3}\pi R_E^3 \rho_t \quad \text{and} \quad V = \frac{4}{3}\pi R_E^3$$

and using (22) we get

$$\frac{dS_I}{dt} = \frac{1}{T_E}[6\pi R_E{}^3 H\rho_D(1+\omega_D)^2 - 4\pi R_E{}^3 H\rho_D(1+\omega_D)]. \tag{22}$$

So combining (25) and (26) the time variation of the total entropy is given by:

$$\frac{d}{dt}(S_E + SI) = 6\pi R_E{}^3 H\frac{\rho_D}{T_E}(1+\omega_D)^2 \tag{23}$$

which is positive definite i.e. the generalized second law of thermodynamics is always valid.

One may note that for $f(R)$ gravity if we proceed in the similar way we have the same conclusion as in (25). Thus we can say that for the universe filled with only holographic dark energy the generalized second law of thermodynamics (GSLT) is always satisfied on the event horizon assuming the validity of the first law of thermodynamics. The calculations show that GSLT do not depend on the modified field equations in these gravity theories but depend on the equation of continuity.

## 4 Generalized Second Law of Thermodynamics of the Universe with Non-Interacting Two Fluid System

In this section we investigate the validity of the generalized second law of thermodynamics of the universe bounded by the event horizon when the matter in the universe is taken in the form of non-interacting two fluid system—one component is the holographic dark energy model and the other component is in the form of dust. The universe is chosen as before to be homogeneous and isotropic and the validity of the first law has also been assumed here.

The modified Friedmann equations in this modified gravity theory are same as (26) and (27) where $\rho$ is replaced by $\rho_t(=\rho_m+\rho_D)$ and $p$ by $p_D$. Here $\rho_D$ and $p_D$ correspond to energy density and thermodynamic pressure of the holographic dark energy model having equation of state $p_D = \omega_D\rho_D$ while $\rho_m$ is the energy density corresponding to dust. As the two component matter system is non interacting so they have separate energy conservation equations namely

$$\dot{\rho_m} + 3H(\rho_m) = 0 \tag{24}$$

and

$$\dot{\rho_D} + 3H(\rho_D + p_D) = 0. \tag{25}$$

As the universe is bounded by the event horizon so the energy density of the holographic model can be written as [38]

$$\rho_D = 3c^2 R_E{}^{-2}. \tag{26}$$

Taking logarithm on both side and differentiating with respect to time [assuming $3c^2 = 1$] we have the small variation of the radius of the horizon as in (22).

The amount of energy crossing the event horizon in time $dt$ has the expression

$$-dE = 4\pi R_E{}^3 H(\rho_t + p_D)dt. \tag{27}$$

Then the validity of the first law of thermodynamics gives

$$\frac{dS_E}{dt} = \frac{4\pi R_E{}^3 H}{T_E}(\rho_t + p_D). \tag{28}$$

Now taking

$$E_I = \frac{4}{3}\pi R_E{}^3 \rho_t \quad \text{and} \quad V = \frac{4}{3}\pi R_E{}^3,$$

and using the equation of continuity (27), the Gibbs equation leads to

$$\frac{dS_I}{dt} = \frac{4\pi R_E{}^3}{T_E} H (\rho_t + p_D) \left[ \frac{3}{2}(1 + \omega_D) - 1 \right] \tag{29}$$

where we have used the expression of $dR_E$ from (26). Hence combining (28) and (29) the resulting change of total entropy is given by

$$\frac{d}{dt}(S_I + S_E) = \frac{6\pi R_E{}^3 H}{T_E}(\rho_t + p_D)(\omega_D + 1). \tag{30}$$

Since in the above calculations we have not used any of the modified field equations, so we have the same result irrespective of any gravity theory i.e. in MGLC, Gauss-Bonnet gravity etc.

## 5 Conclusions

In the present work we examine the validity of the generalized second law of thermodynamics on the event horizon assuming the validity of the first law of thermodynamics in different gravity theories in a general way. We consider the universe as a thermodynamical system and is filled with perfect fluid, only holographic dark energy and non-interacting two fluids in Sects. 2, 3 and 4 respectively. From the above studies we can draw the following general conclusions in different gravity theories:

I. If $R_A$ and $R_H$ denote the radius of the apparent horizon and the Hubble horizon then the relation between the horizons are the following [54]:

$$k = 0 \text{ (Flat model)}: \quad R_A = \frac{1}{H} = R_H < R_E,$$

$$k = -1 \text{ (Open model)}: \quad R_H < R_A < R_E,$$

$$k = -1 \text{ (Closed model)}: \quad R_A < R_E < R_H \quad \text{or} \quad R_A < R_H < R_E.$$

In Sect. 2 from (11) (or (18)) we can conclude that the generalized second law of thermodynamics (GSLT) holds at the event horizon starting from the first law of thermodynamics provided the weak energy condition is satisfied for flat and open model of the universe. However, for closed FRW model, GSLT is satisfied and $R_H < R_E$ or there is a violation of weak energy condition (exotic matter) and $R_E < R_H$.

II. It is to be noted that in reference [54] similar conclusion was drawn for Einstein Gravity as well as for Einstein-Gauss-Bonnet (EGB) gravity.

III. When holographic dark energy is taken as the matter in the flat FRW universe in Sect. 3 we have seen that the generalized second law of thermodynamics holds on the event horizon starting from the validity of the first law for MGLC and $f(R)$ gravity. Here we need no restriction either on the matter or on the geometry.

IV. Also from Sect. 3 one can check that if we use the Gauss-Bonnet theory as a particular case of $f(R)$ gravity then the Einstein field equation as well as the equations for the thermodynamical studies are very similar to those that we have presented for MGLC provided the Gauss-Bonnet coupling parameter ($\tilde{\alpha}$) is related to the dimension less parameter $\alpha$ in LQG by the relation $\tilde{\alpha} = \frac{\alpha H^2}{8\pi^2}$.

V. In Sect. 4 for non-interacting two fluid system, if the holographic dark energy component individually satisfies the weak energy condition i.e. the HDE component is not of the phantom nature then universe as a thermodynamical system with matter in the form of non-interacting two fluid system always obey the GSLT in the two gravity theory we are considering. One may note that identical conclusion was obtained in ref [55] for Einstein gravity (see also[1] for EGB gravity).

VI. One thing may be noted that throughout the paper we have not used any explicit form of temperature and entropy on the event horizon. Also we are considering the universe is in thermodynamical equilibrium so the temperature on the event horizon is similar with the temperature inside the event horizon.

VII. To examine the validity of the GSLT on the event horizon, we have not used any modified Friedmann equations for the gravity theories (those we are considering)—only the continuity equation is needed. As Einstein field equations and the first law of thermodynamics on the apparent horizon are equivalent so we may speculate that the validity of the thermodynamical laws on the event horizon does not depend on the validity of the laws at the apparent horizon.

VIII. Finally, we may conclude that if we start with the first law of thermodynamics on the event horizon then the conditions for validity of GSLT does not depend on any specific gravity theory (discussed here). Also the conclusion is not affected by the matter that we have considered—the only restriction is that any component of the matter should not be of phantom nature.

For further work, we study the validity of GSLT on the event horizon for other gravity theories and examine whether any general conclusion independent of any specific gravity theory can be drawn. Also thermodynamical laws in phantom era may have distinct features.

## References

1. Jacobson, T.: Phys. Rev. Lett. **75**, 1260 (1995)
2. Bekenstein, J.D.: Phys. Rev. D **7**, 2333 (1973)
3. Padmanavan, T.: Class. Quantum Gravity **19**, 5387 (2002)
4. Cai, R.G., Kim, S.P.: J. High Energy Phys. **02**, 050 (2005)
5. Perimutter, S., et al. (Supernova Cosmology Project Collaboration): Astrophys. J. **517**, 565 (1999)
6. Riess, A.G., et al. (Supernova Search Team Collaboration): Astron. J. **116**, 1009 (1998)
7. Astier, P., et al. (The SNLS Collaboration): Astron. Astrophys. **447**, 31 (2006)
8. Riess, A.G., et al.: Astrophys. J. **659**, 98 (2007)
9. Spergel, D.N., et al. (WMAP Collaboration): Astrophys. J. Suppl. Ser. **148**, 175 (2003)
10. Peiris, H.V., et al. (WMAP Collaboration): Astrophys. J. Suppl. Ser. **148**, 213 (2003)
11. Spergel, D.N., et al. (WMAP Collaboration): Astrophys. J. Suppl. Ser. **170**, 377 (2007)
12. Komatsu, E., et al., (WMAP Collaboration): arXiv:0803.0547 [astro-ph]
13. Sahi, V.: AIP Conf. Proc. **782**, 166 (2005) [J. Phys. Conf. Ser. **31**, 115 (2006)]

---

[1]In our earlier work [55] we have studied the generalized second law of thermodynamics as in Sect. 4 for Gauss-Bonnet gravity but we cannot make any definite conclusion due to complicated expression for the rate of change of total entropy. In the present work we can simplify that expression further and we have the same conclusion as conclusion (Sect. 5).

14. Padmanavan, T.: Phys. Rep. **380**, 235 (2002)
15. Copeland, E.J., Sami, M., Tsujikawa, S.: Int. J. Mod. Phys. D **15**, 1753 (2006)
16. Durrer, R., Marteens, R.: Gen. Relativ. Gravit. **40**, 301 (2008)
17. Nojiri, S., Odintsov, S.D.: Int. J. Geom. Methods Mod. Phys. **4**, 115 (2007)
18. Nojiri, S., Odintsov, S.: arXiv:0801.4843 [astro-ph]
19. Nojiri, S., Odintsov, S.: arXiv:0807.0685 [hep-th ]
20. Capozziello, S.: Int. J. Mod. Phys. D **11**, 483 (2002)
21. Carroll, S.M., Duvvuri, V., Trodden, M., Turner, M.S.: Phys. Rev. D **68**, 043528 (2004)
22. Nojiri, S., Odintsov, S.D.: Phys. Rev. D **68**, 123512 (2003)
23. Nojiri, S., Odintsov, S.D.: Phys. Rev. D **74**, 086005 (2006)
24. Bamba, K., Geng, C.-Q.: arXiv:0901.1509 [hep-th]
25. Sadjadi, H.M.: Phys. Rev. D **76**, 104024 (2007)
26. Rovelli, C.: Phys. Rev. Lett. **77**, 3288 (1996)
27. Ashtekar, A., Baez, J., Corichi, A., Krasnov, K.: Phys. Rev. Lett. **80**, 904 (1998)
28. Paul, R.K., Majumder, P.: Phys. Rev. Lett. **84**, 5255 (2000)
29. Ghosh, A., Mitra, P.: Phys. Rev. D **71**, 027502 (2005)
30. Domagala, M., Lewandowski, J.: Class. Quantum Gravity **21**, 5233 (2004)
31. Meissner, K.A.: Class. Quantum Gravity **21**, 5245 (2004)
32. Hod, S.: Class. Quantum Gravity **21**, L97 (2004)
33. Medved, A.J.M.: Class. Quantum Gravity **22**, 133 (2005)
34. Chatterjee, A., Mazumder, P.: arXiv:gr-qc/0303030
35. Ashtekar, A., Pawlowski, T., Sing, P.: Phys. Rev. Lett. **96**, 141301 (2006)
36. Ashtekar, A., Pawlowski, T., Sing, P.: Phys. Rev. D **74**, 084003 (2006)
37. Ashtekar, A., Pawlowski, T., Sing, P., Vandersloot, K.: Phys. Rev. D **75**, 024035 (2007)
38. Wang, B., Gong, Y., Abdalla, E.: Phys. Rev. D **74**, 083520 (2006)
39. Li, M.: Phys. Lett. B **603**, 01 (2004)
40. Setare, M.R., Shafei, S.: J. Cosmol. Astropart. Phys. **0609**, 011 (2006). arXiv:gr-qc/0606103
41. Hu, B., Ling, Y.: Phys. Rev. D **73**, 123510 (2006)
42. Ito, M.: Europhys. Lett. **71**, 712 (2005)
43. Nojiri, S., Odintsov, S.: Gen. Relativ. Gravit. **38**, 1285 (2006)
44. Saridakis, E.N.: Phys. Lett. B **660**, 138 (2008)
45. Huang, Q.G., Li, M.: J. Cosmol. Astropart. Phys. **0408**, 013 (2004)
46. Zhang, X.: IJMPD **14**, 1597 (2005)
47. Pavon, D., Zimdahl, W.: Phys. Lett. B **628**, 206 (2005)
48. Kim, H., Lee, H.W., Myung, Y.S.: Phys. Lett. B **632**, 605 (2006)
49. Hsu, S.D.: Phys. Lett. B **594**, 01 (2004)
50. Horvat, R.: Phys. Rev. D **70**, 087301 (2004)
51. Cai, R.G., Cao, L.M., Hu, Y.P.: arXiv:0807.1232 [hep-th]
52. Bousso, R.S.: Phys. Rev. D **71**, 064024 (2005)
53. Izquierdo, G., Pavon, D.: Phys. Lett. B **633**, 420 (2006)
54. Mazumder, N., Chakraborty, S.: Class. Quantum Gravity **26**, 195016 (2009)
55. Mazumder, N., Chakraborty, S.: Gen. Relativ. Gravit. (2009). doi:10.1007/s10714-009-0881-z